Simulated Annealing: A Monte Carlo Method for GPS Surveying

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Abstract. This paper describes simulated annealing technique,which is a Monte Carlo method, to analyze and improve the efficiency of the design of Global Positioning System (GPS) surveying networks. The paper proposes various local search procedures which can be coupled with the simulated annealing technique.

1 Introduction

The GPS is a satellite-based navigation system that permit users to determine their three-dimensional position. Sequencing play a crucial role in GPS management. In the last decades simulated annealing is considered as a heuristic method that uses a Monte Carlo global minimization technique for minimizing multi-variance functions [2]. The concept is based on the manner in which liquids freeze or metal recrystallize in the process of annealing. When related to the positioning the process of designing a GPS surveying network can be described as follows. A number of receivers r are placed at stations n and take simultaneous measurements from satellites. The receivers are then moved to other stations for further measurements. The problem is to search for a best order in which to consecutively observe these sessions to have the best schedule at minimum cost (time) i.e.: $Minimize: \quad C(V) = \sum_{p \in R} C(S_p)$, where: $C(V)$ is the total cost of a feasible schedule V; S_p is the rout of the receiver p in a schedule; R is the set of receivers.

To maximize the benefit to use simulated annealing, different Local Search (LS) procedures have been coupled with this technique to improve the performance and explore the search space more effectively. The rest of this paper is organized as follows. Section 2 outlines the simulated annealing method applied to GPS surveying networks. The search strategy of the local search is explained in Section 3 and followed by different local search procedures. Several case studies and the obtained numerical results are reported in Section 4.

2 Simulated Annealing

Simulated annealing is a flexible and robust technique which derived from physical science. This method has proved to be a local search method and can be successfully applied to the majority of real-life problems [1–3]. The algorithm starts by generating an initial solution and by initializing the so-called temperature parameter T. The temperature is decreased during the search process, thus at the beginning of the search the probability of accepting uphill moves is high and it gradually decreases. Initial solution is a random solution. The structure of the simulated annealing algorithm is shown below:

$$\textbf{Step1}. \begin{cases} \text{Initializing the cooling parameters:} \\ \text{Set the initial starting value of the temperature parameter, } T > 0; \\ \text{Set the temperature length, } L; \\ \text{Set the cooling ratio, } F; \\ \text{Set the number of iterations, } K = 0 \end{cases}$$

$$\textbf{Step2}. \begin{cases} \text{Select a neighbor } V' \text{ of } V \text{ where } V' \in I(V) \\ \text{Let } C(V') = \text{the cost of the schedule } V' \\ \text{Compute the move value } \Delta = C(V') - C(V) \end{cases}$$

$$\textbf{Step3}. \begin{cases} \text{If } \Delta \leq 0 \text{ accept } V' \text{ as a new solution and set } V = V' \\ \text{ELSE} \\ \text{IF } e^{\Delta/T} > \Theta \text{ set } V = V', \\ \text{where } \Theta \text{ is a uniform random number } 0 < \Theta < 1 \\ \text{OTHERWISE retain the current solution } V. \end{cases}$$

$$\textbf{Step4}. \begin{cases} \text{Updating the annealing parameters using the} \\ \text{cooling schedule } T_{k+1} = F * T_k \; k = \{1, 2, \ldots\} \end{cases}$$

Step 5. IF the stopping criteria is met THEN

$$\textbf{Step6}. \begin{cases} \text{Show the output} \\ \text{Declare the best solution} \\ \text{OTHERWISE Go to step 4.} \end{cases}$$

3 Local Search Strategies

LS procedure perturb given solution to generate different neighborhoods using a move generation mechanism [6]. A move generation is a transition from a schedule V to another one $V' \in I(V)$ in one step (iteration). In Saleh [4], a local search procedure that satisfies the GPS requirements has been developed. This procedure is based on the sequential session-interchange. The potential pair-swaps are examined in the order $(1,2)$,$(1,3)$, …, $(1,n)$,$(2,3)$, $(2,4)$, …, $(n-1,n)$, n is the number of sessions. The solution is represented by graph. The nodes correspond to the sessions and the edges correspond to the observation order. For comparison reason this sequential local search procedure is called procedure (a). In this paper two new local search procedures have been developed. In the first, see Figure 1, A and B are chains of nodes (part of the solution) while $\{0,i\}$ and $\{i+1,n\}$ are, the first and the last nodes of the chains A and B respectively. In this procedure only one exchange of new edge $[n-1,0]$ has been performed each iteration and this can be done by selecting one edge $[i,i+1]$ to be removed. For comparison reason this local search procedure is called (b1).

In the second procedure, as shown in Figure 2, A, B and C are chains of nodes (part of the solution), where $\{0,i\}$, $\{i+1,j\}$ and $\{j+1,n-1\}$ are the

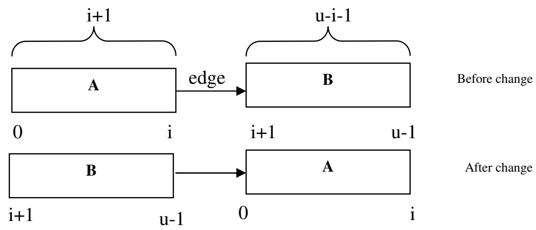

Fig. 1. Local search structure (b1) with 1-exchange of edges.

first and the last nodes of the chains A, B and C respectively. In the case of two edge exchange, there are several possibilities to build a new solution where the edges $[i, i+1]$ and $[j, j+1]$ are selected to be removed in one iteration. For comparison reason this local search procedure is called (b2).

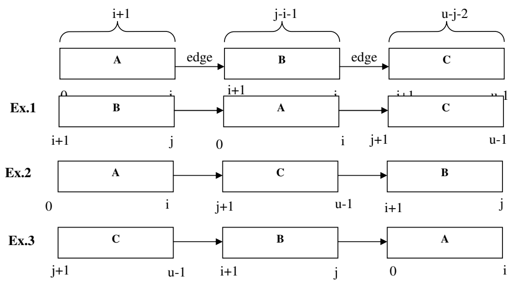

Fig. 2. Local search structure (b2) with 2-exchange of edges.

4 Experimental Results

This section reports on the computational experience of the simulated annealing coupled with local search procedures using real GPS networks. The first network is a GPS network of Malta. The initial schedule with a cost of 1405 minutes was composed of 38 sessions observed. The second network is a GPS network of Seychelles. The initial schedule with cost of 994 minutes was composed of 71 sessions observed.

Where: C_{INT} - The cost of the initial solution; C_a - Schedule using local search procedure (a); C_{b1} - Schedule using local search procedure (b1); C_{b2} Schedule using local search procedure (b2).

We compare simulated annealing algorithm coupled with Saleh and Dare [4, 5] LS procedure, with developed in this paper LS procedures (b1) and (b2). The analysis of the results in Table 1 shows the advantage of the local search procedure (b2). We use the following parameters: initial temperature 8, final temperature 3, cooling parameter 0.9 for all tests. In the simulated annealing the selected set of neighbors has an important role in achieving good results.

Table 1. Comparison of local search techniques applied to different types of GPS networks

Data	Malta	improvement	Seychelles	improvement
n	38		71	
C_{INT}	1405	0%	994	0%
C_a	1375	2.14%	969	2.52%
C_{b1}	1285	8.54%	994	0%
C_{b2}	1105	21.34%	707	28.87%

The size of this selected set is larger in the procedure (b2) with comparison to the other two procedures.

5 Conclusion

In this paper two local search procedures have been developed and compared with local search procedure from [4, 5], procedure (a). The comparison of the performance of the simulated annealing of these procedures applying to different GPS networks is reported. The obtained results are encouraging and the ability of the developed techniques to generate rapidly high-quality solutions for observing GPS networks can be seen. The problem is important because it arises in mobile phone communications too.

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